STATISTICAL MODEL OF TURBULENT MIXING OF RELAXING GASES BEHIND AN ARRAY OF SMALL-SCALE NOZZLES

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An approach to the description of mixing of the relaxing gases behind an array of small-scale nozzles is proposed, on the basis of a spectral model of turbulence and the density function of the probability distribution for the concentration.

In any practical applications, it is necessary to calculate the flow behind the plane of an array consisting of a large number of small-scale nozzles (Fig. 1). Such arrays are usually employed for the rapid mixing of various gases and the preparation of a homogeneous medium with specified characteristics. As a rule, because of the velocity difference of the adjacent jets, the flow behind the plane of the nozzle array is turbulent and may be described by a system of three-dimensional dynamic Reynolds equations [1]. Solving such a system of equations using even the simplest model of turbulence is very complex. The problem becomes even more complex on taking account of nonequilibrium excitation of internal degrees of freedom of the molecule, with subsequent relaxation. This must be taken into account in considering flow with large gradients of the gas-dynamic parameters, for example, in gas-dynamic [2] and chemical [3] lasers, including gas-dynamic lasers with mixing (GDLM) [4].

In analyzing GDLM, it is usual to make a series of assumptions leading to a simple model of mixing.

The model of "instantaneous" mixing, based on integral relations expressing the conversion of mass, momentum, and energy, permits quantitative estimation of the gas-dynamic parameters of mixing at the molecular level of the flow and, in some cases (when the characteristic time of mass transfer is less than the relaxation time), satisfactory description of the distribution of laser characteristics of the medium (amplification factor, specific energy reserve) downstream from the nozzle array [5]. However, in such an integral approach, the dynamics of flow transition from the initial to the final state is unknown (even through these states are completely determined). Therefore, in those cases where the gas-dynamic parameters of the final state differ significantly from the mean parameters at the plane of the nozzle array, the model of "instantaneous" mixing is unsatisfactory, since it is necessary to take account of the change in mean quantities because of breakdown of their pulsational fields in the mixing process.

Attempts to take account of mixing dynamics on the basis of the model of "open current tubes" were made in [6-8]. In this model, the transverse mass transfer is taken phenomenologically into account, as well as the momentum and energy transfer, by introducing two initially unknown functions $G_i(x)$, which are the flow rates of the pure components in the mixing layer. Determining the form of these functions on the basis of any additional information is the basic problem in using the given model. As shown in [8], on specifying $G_i(x)$ in the form of linear (or other simple) functions of the longitudinal coordinate x, when the number of arbitrary parameters is a minimum, the quantitative agreement of the results of calculating the laser characteristic with the experimental data is not satisfactory.

The basic deficiency of these approaches to the calculation of laser characteristics of supersonic relaxing jets is that they take no account of the delay in mixing to the molecular level due to the complex evolution of the initially large moles of pure component in the turbulent velocity field and the finite rate of molecular diffusion. This delay of mixing

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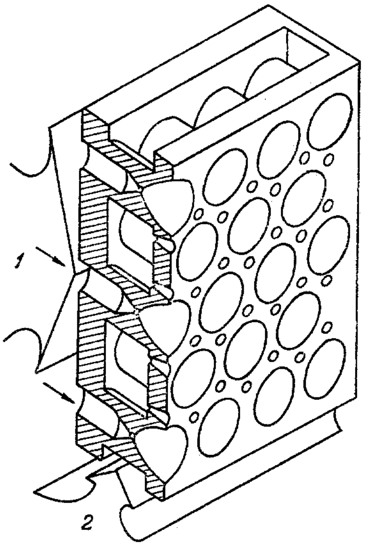

Fig. 1

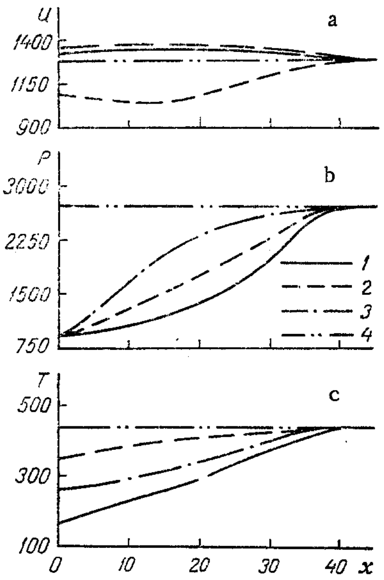

Fig. 2

Fig. 1. Mixing array of small-scale nozzles.

Fig. 2. Calculation of mean velocity u, m/sec (a), pressure P, Pa (b), and temperature T, K (c) behind the plane of the nozzle array in the mixing of nonrelaxing jets N_2–N_2: 1) by the present model with $\varphi = x/L_2$, $\psi = 1 - x/L_2$; 2, 3) by the model of [8] with $L_2/L_1 = 1$ and 2.5, respectively ($L_2 = 40$ cm); 4) by the model of "instantaneous" mixing.

in flows with vibrational and chemical disequilibrium was taken into account in [9-11], for example, on the basis of the model of "nonmixing" and "damping" of the reaction rate constants, by multiplying them by a "nonmixing" function which depends on the mean square pulsations of the concentration.

In the present work, an approach to calculating mixing and relaxation processes behind an array of small-scale nozzles is developed, using the density function of the probability distribution (DFPD) of the scalar field and the equations of turbulent mixing for the distribution function of the intensity of velocity and concentration pulsations over the spectrum of length scales.

With all the complexity of the turbulent-mixing process, it may be noted that the flow behind the plane of the nozzle array is characterized by the presence of macromoles of the pure components and regions with a homogeneous composition in which the components are already mixed to the molecular level. Introducing the functions $G_i(x)$ describing the mass flow rates of the components mixed to the molecular level, the following obvious relation holds

$$G_i(x) = \varphi(x) G_i^0, \quad i = 1, 2, \tag{1}$$

where G_i^0 is the total flow rate of the i-th component; $\varphi(x)$ is the proportion of components mixed to the molecular level.

Writing one-dimensional steady Euler equations for each component of the medium, under the assumption that the transverse pressure gradient is zero, and introducing phenomenologically, according to Eq. (1), the mass transfer between the components, the following initial system of dynamic equations is obtained

$$\frac{1}{\rho_i}\frac{d\rho_i}{dx} + \frac{1}{u_i}\frac{du_i}{dx} + \frac{1}{A_i}\frac{dA_i}{dx} = -\frac{1}{1-\varphi}\frac{d\varphi}{dx}; \tag{2}$$

$$\rho_i u_i \frac{du_i}{dx} + \frac{dP}{dx} = 0; \tag{3}$$

$$\frac{dH_i}{dx} + u_i \frac{du_i}{dx} = 0; \tag{4}$$

$$\frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{u}\frac{du}{dx} + \frac{1}{A}\frac{dA}{dx} = \frac{1}{\varphi}\frac{d\varphi}{dx}; \tag{5}$$

$$\rho u \frac{du}{dx} + \frac{dP}{dx} = \rho u Q_u; \tag{6}$$

$$\frac{dH}{dx} + u\frac{du}{dx} + \frac{d\psi}{dx}\frac{\overline{u'^2(0)}}{2} = 0; \tag{7}$$

$$p = \frac{R\rho_i T_i}{\mu^{(i)}} = \frac{R\rho T}{\mu}, \tag{8}$$

where

$$H = \frac{R}{\mu}\left(\beta_0 T + \sum_{j=1}^{N_J} \xi_j \Theta_j e_j\right); \quad A_i = \frac{(1-\varphi)G_i^0}{\rho_i u_i}; \quad A = \frac{\varphi(1+\delta_G)G_2^0}{\rho u};$$

$$Q_u = \frac{1}{\varphi}\frac{d\varphi}{dx}[(u_1 - u)\delta_G + u_2 - u]; \quad \delta_G = \frac{G_1^0}{G_2^0};$$

$$e_j = \frac{g_j}{\exp(\Theta_j/T_j) - 1}; \quad \beta_0 = \frac{5}{2} + \frac{3}{2}\xi_N + \xi_L.$$

Quantities with no subscript i correspond to regions of the flow where the components are mixed to the molecular level. In the equation for the total enthalpy – Eq. (7) – in contrast to the analogous equation of [6-8], there is an additional term of the form $(d\psi/dx)\cdot\overline{u'^2(0)}/2$, refining the energy structure of the flow behind the plane of the nozzle array. The mean mass velocity at the onset of mixing is defined as

$$u(0) = \frac{u_1^0 \delta_G + u_2^0}{1 + \delta_G}. \tag{9}$$

The mean square velocity pulsations relative to the mean mass level may be written in the following form, taking account of Eq. (9)

$$\overline{u'^2(0)} = \frac{(u_1^0 - u_2^0)^2 \delta_G}{(1 + \delta_G)^2}. \tag{10}$$

It follows from Eq. (10) that, when $\delta_G \sim 1$, and with a sufficiently large velocity difference of the flows at the array cross section, the energy of pulsations of the velocity field cannot be neglected, since it becomes comparable with the kinetic energy of the mean motion. Hence, the total enthalpy of the flow must include three terms: H, the mean specific enthalpy; $u^2/2$, the kinetic energy of mean motion; $\overline{u'^2}/2$, the energy of turbulent pulsations of the velocity field. The function $\psi(x) = \overline{u'^2(x)}/\overline{u'^2(0)}$ introduced in Eq. (7) characterizes the intensity of the turbulent velocity field. After reducing to normal form and adding kinetic equations, the system in Eq. (2)-(8) may be written in the form

$$\frac{dP}{dx} = -\frac{P}{S_A}\left[\frac{dA_0}{dx} + S_p + S_G\right]; \tag{11}$$

$$\frac{du_i}{dx} = -\frac{\lambda_i u_i}{P}\frac{dP}{dx}; \tag{12}$$

$$\frac{dT_i}{dx} = T_i\left[\frac{1}{\beta_i P}\frac{dP}{dx} - S_T^{(i)}\right]; \tag{13}$$

$$\frac{du}{dx} = -\frac{\lambda u}{P}\frac{dP}{dx} + Q_u; \tag{14}$$

$$\frac{dT}{dx} = T\left[\frac{1}{\beta P}\frac{dP}{dx} - S_T - S_u - S_\psi\right]; \tag{15}$$

$$\frac{de_j}{dx} = \varphi\left(\frac{de_j}{dx}\right)_{\text{rel}} + Q_{ej}; \tag{16}$$

$$\frac{de_j^{(i)}}{dx} = (1-\varphi)\frac{(m\delta_G)^{2-i}}{1+m\delta_G}\left(\frac{de_j^{(i)}}{dx}\right)_{\text{rel}}, \quad i = 1,\ 2; \quad j = 1,\ \ldots,\ N_J; \tag{17}$$

where

$$Q_{ej} = \frac{1}{\varphi}\,\frac{d\varphi}{dx}\,\frac{1}{\xi_j^{(2)} + m\delta_G\,\xi_j^{(i)}}\,[(e_j^{(1)} - e_j)\,\xi^{(1)}m\delta_G + (e_j^{(2)} - e_j)\,\xi_j^{(2)}];$$

$$S_p = \sum_{i=1}^{2} A_i S_{\text{T}}^{(i)} + A\,[S_{\text{T}} + S_u\,(1+\beta\lambda) + S_\psi];$$

$$S_A = A_1\left(1 - \lambda_1 - \frac{1}{\beta_1}\right) + A_2\left(1 - \lambda_2 - \frac{1}{\beta_2}\right) + A\left(1 - \lambda - \frac{1}{\beta}\right);$$

$$S_G = \frac{d\varphi}{dx}\,\frac{G_2^0}{P}\,[(\lambda_1 u_1 - \lambda u)\,\delta_G + \lambda_2 u_2 - \lambda u]; \quad A_0 = A_1 + A_2 + A;$$

$$S_{\text{T}} = \frac{1}{\beta T}\sum_{j=1}^{N_J} \xi_j \Theta_j \frac{de_j}{dx}; \quad S_\psi = \frac{1}{\beta\lambda u^2}\,\frac{\overline{u'^2(0)}}{2}\,\frac{d\psi}{dx};$$

$$S_u = \frac{Q_u}{\beta\lambda u}; \quad \beta = \beta_0 + \sum_{j\in\Omega} \xi_j \Theta_j \frac{\overline{de_j}}{dT}; \quad \lambda = \frac{RT}{\mu u^2};$$

$$m = \frac{\mu^{(2)}}{\mu^{(1)}}; \quad \bar{e}_j = \frac{g_j}{\exp(\Theta/T) - 1};$$

Ω is the set of indices of the vibrational modes of the mixture in equilibrium with the translational temperature; $(de_j/dx)_{\text{rel}}$ is the relaxational component of the kinetic equations, in which the constants of V–V, V–T, and V–V' exchange from [12, 13] are employed.

To compare the gas-dynamic model obtained in Eqs. (11)-(17) with the model of [8], the mixing of nonrelaxing jets of hot (T_{02} = 2000 K, P_{02} = 1.0 MPa) and cold (T_{01} = 300 K, P_{01} = 0.9 MPa) nitrogen is calculated, with the following parameters in the plane of the nozzle array: T_1 = 58.8 K, P_1 = 3206 Pa, u_1 = 698 m/sec, T_2 = 292 K, P_2 = 1036 Pa, u_2 = 1918 m/sec. After calculating the shock-wave interaction and additional expansion of the jets by the method described in [7], the gas parameters are as follows: T_1 = 41.3 K, T_2 = 331 K, u_1 = 723 m/sec, u_2 = 1897 m/sec, $P_1 = P_2$ = 898 Pa.

The results of calculating the mean gas-dynamic parameters of the flow by the "instantaneous"-mixing model [8] with two values of the ratio of the disappearance lengths of the cores of pure components L_2/L_1 and the model in Eqs. (11)-(17) with linear dependences $\varphi(x) = x/L_2$, $\psi(x) = 1 - x/L_2$ are shown in Fig. 2. It is evident that, when $L_1 = L_2$, the model of [8] is closest to the present model in describing the distribution of the mean velocity and pressure in the flow, but the mean temperature differs very significantly here. When L_2/L_1 = 2.5, the difference in the mean-temperature distributions decreases, but at the same time becomes larger for the mean velocity and pressure. Thus, Eq. (7) and the proportion of mixed components in the form in Eq. (1) significantly influence the distribution of mean gas-dynamic characteristics of the flow, and the functions $\varphi(x)$ and $\psi(x)$ must be found on the basis of a more adequate model of turbulent mixing if they are to be satisfactorily described.

In the initial section of the flow behind the plane of the mixing array (Fig. 1), the turbulence is significantly nonequilibrium in type. Disequilibrium appears in the lack of development of the intensity spectra of the turbulent pulsations of velocity and concentration with respect to the length scale and also in the pronounced deviation of the single-point distribution functions of the concentration [14] and velocity from Gaussian form. At the same time, second-order single-point models (of type k–ε) describe mixing processes under the assumption of equilibrium of all the turbulence characteristics [15-17]. In addition, these models bear no information on the proportion of the components mixed to the molecular level, which is an important characteristic for relaxational processes occurring as a result of the molecular collisions. Therefore, for the correct modeling of relaxation

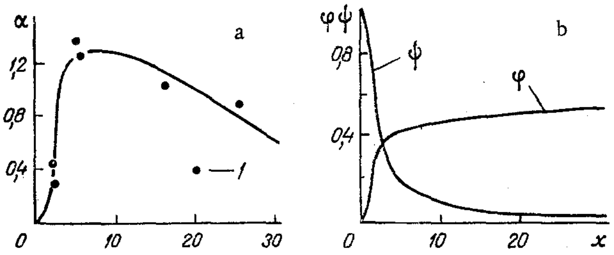

Fig. 3. Comparison of amplification factor $\overline{\alpha}$, m^{-1}, calculated by the present model (continuous curve) and experimental data [27] (a), and variation in φ and ψ along the longitudinal coordinate x, cm, behind the plane of a nozzle array (b); 1) experiment.

processes, the minimum possible level of description of turbulent mixing is the use of two-point moment characteristics of turbulence and single-point probability densities. Accordingly, it is important to consider two aspects in the interaction: the dynamics of the intensity distribution of the pulsational concentration field with respect to the spectrum of length scales and the dynamics of the concentration pulsations. The first aspect is determined by the turbulent small-scale motion of the gases being mixed; the second is associated with the action of molecular diffusion; they interact because the evolution of the intensity distribution of the concentration pulsations with respect to the length scale creates variable boundary conditions for the molecular diffusion. In [18], equations were given for the spectral distributions $P_t(r)$ and $P_t^c(r)$

$$\frac{\partial P_t(r)}{\partial t}=2\gamma\sqrt{rP_t(r)}\left[\frac{\partial P_t(r)}{\partial r}+\frac{4}{r}P_t(r)\right]+ \\ +2\left[\nu+\gamma\int_0^r\sqrt{r'P_t(r')}\,dr'\right]\left[\frac{\partial^2 P_t(r)}{\partial r^2}+\frac{4}{r}\frac{\partial P_t(r)}{\partial r}-\frac{4}{r^2}P_t(r)\right]; \tag{18}$$

$$\frac{\partial P_t^c(r)}{\partial t}=2\beta\sqrt{rP_t(r)}\left[\frac{\partial P_t^c(r)}{\partial r}+\frac{2}{r}P_t^c(r)\right]+ \\ +2\left[\chi+\beta\int_0^r\sqrt{r'P_t(r')}\,dr'\right]\left[\frac{\partial^2 P_t^c(r)}{\partial r^2}+\frac{2}{r}\frac{\partial P_t^c(r)}{\partial r}\frac{2}{r^2}P_t^c(r)\right], \tag{19}$$

where ν, χ are the mean kinematic viscosity and diffusion coefficient; γ, β are constants determined from experiment. The system in Eqs. (18)-(19) describes the mixing of the components in an isotropic turbulent flow, in which turbulent motion is small-scale, while the medium is homogeneous on average and mixed to macromoles of the mixture. In fact, for arrays with a characteristic linear dimension $L_m \sim 10^{-1}...1$ m, individual-nozzle dimensions $r_0 \sim 10^{-3}$ m, mean flow velocity $U \sim 10^3$ m/sec, and velocity difference of adjacent jets $|u_1^0 - u_2^0| \sim 10^2\text{-}10^3$ m/sec

$$r_0 \ll L_m,\quad \tau \ll T_m, \tag{20}$$

where $T_m = L_m/U \sim 10^{-4}\text{-}10^{-3}$ sec; $\tau = r_0/|u_1^0 - u_2^0| \sim 10^{-6}\text{-}10^{-5}$ sec.

The Reynolds number of the flow overall when $\nu \sim 10^{-3}\text{-}10^{-4}$ m²/sec is large ($Re = UL_m/\nu \sim 10^5\text{-}10^7$); therefore, it is natural to suppose that, in sufficiently small regions of the flow with a dimension of order r_0, the hypothesis of locally isotropic turbulence may be adopted with good approximation [19] and, in describing such flow, it is natural to use the apparatus developed in the theory of isotropic turbulence. In some cases, the most significant effects of inhomogeneity may be taken into account using the model of [18] with a mean velocity, density, or any other parameter which varies over time [20, 21]. Solving Eq. (18) for $P_t(r)$, the expression for $\psi(x)$ may be written in the form

$$\psi(x)=\frac{\overline{u'^2(x)}}{\overline{u'^2(0)}}=\frac{1}{\overline{u'^2(0)}}\int_0^\infty P_t(r)\,dr, \tag{21}$$

where the relation between the coordinate and the time is

$$dt = \frac{dx}{u(x)}. \tag{22}$$

In determining the proportion of components mixed to the molecular level $\Phi(x)$, it is necessary to use the DFPD of the concentration field. As a rule, in calculating the DFPD, the presence of the spectrum $P_t^c(r)$ is not taken into account. The state of the concentration field in this respect is taken into account in terms of the mean length scale or the mean dissipation rate of the intensity of the concentration pulsations. However, if the multiplicity of scales of the mixing process is disregarded, perceptible errors may result [22]. The equation for a single-point DFPD of the concentration [23] in the case of an isotropic single-scale pulsational field may be reduced to the form [24]

$$\frac{\partial f_t(c', r)}{\partial t} = \frac{3\chi}{r^2} \frac{\partial}{\partial c'} [c' f_t(c', r)]. \tag{23}$$

A closed equation of analogous form was obtained in [25] by the method of the linear mean-square approximation (LSME). The solution of Eq. (23) takes the form

$$f_t(c') = e^{3\chi t/r^2} f_0 (c' e^{3\chi t/r^2}). \tag{24}$$

The function in Eq. (24) is a nonrealistic model of turbulent mixing, since at any time it depends on the form of the initial distribution f_0, which contradicts experience. This reveals a serious deficiency of the LSME method: It incorrectly describes small-scale mixing. Taking into account that the mixing process has a multiplicity of scales corrects this significant deficiency. The expression for $f_t(c')$ taking account of the correlation of the dynamics of the intensity distribution of the concentration pulsations with the process of molecular diffusion is as follows

$$f_t(c') = \int_0^\infty e^{3\chi t/r^2} f_0 (c' e^{3\chi t/r^2}) R_t^c (r) dr. \tag{25}$$

Here the solution in Eq. (24) is averaged over the probability distribution of the scales $R_t^c(r)$. Equation (25), written in terms of the spectrum $P_t^c(r)$, takes the form

$$f_t(c') = \int_{\lambda_0}^\infty e^{9\chi t/r^2} f_0 (c' e^{3\chi t/r^2}) P_t^c (r)\, dr, \tag{26}$$

where λ_0 is the scale characterizing the dimension of the flow region in which the components are mixed to the molecular level. The scale λ_0 at each instant of time is less than the microscale of turbulence for the concentration field and is found from the integral equation

$$\int_{\lambda_0}^\infty e^{6\chi t/r^2} P_t^c (r)\, dr = 1. \tag{27}$$

The initial DFPD of the concentration when only pure components are present in the flow is usually chosen in the form of a sum of two δ functions with some weighting factors (for example, $n = \delta_G$)

$$f_0(c) = \frac{n}{1+n} \delta (c - c_1) + \frac{1}{1+n} \delta (c - c_2). \tag{28}$$

The mean concentration is constant for the whole flow

$$\bar{c} = \int_{c_1}^{c_2} f_0 (c)\, dc = \frac{nc_1 + c_2}{1+n}. \tag{29}$$

Introducing the variable $c' = c - \bar{c}$, an expression is obtained for the DFPD of the fluctuational field c' initially

$$f_0 (c') = \frac{n}{1+n} \delta \left[c' + \frac{c_2 - c_1}{1+n}\right] + \frac{1}{1+n} \delta \left[c' - \frac{n (c_2 - c_1)}{1+n}\right]. \tag{30}$$

The proportion of components $\varphi(x)$ mixed to the molecular level, expressed in terms of the DFPD of the concentration pulsations, is determined as follows

$$\varphi(x)=1-\frac{\int\limits_{+\Delta}^{c_2-\bar{c}} c'f_t(c')\,dc'}{\int\limits_{0}^{c_2-\bar{c}} c'f_0(c')\,dc'}=1-\frac{\int\limits_{c_1-\bar{c}}^{-\Delta} c'f_t(c')\,dc'}{\int\limits_{c_1-\bar{c}}^{0} c'f_0(c')\,dc'}, \tag{31}$$

where $\Delta > 0$, $\Delta \to 0$. If the function $P_0{}^c(r)$ is not normalized to unity initially ($\lambda_0 = 0$, i.e., only pure components are present in the flow), then Eq. (31) may be reduced to the following simple form, taking account of Eq. (26) and (30)

$$\varphi(x)=1-\int\limits_{\lambda_0}^{\infty} e^{3\chi t/r^2} P_t^c(r)\,dr. \tag{32}$$

The relations obtained for $\varphi(x)$ and $\psi(x)$ determine the intensity of the turbulent velocity field and the mixing of the flow to the molecular level in the gas-dynamic and kinetic equations. The degree of mixing of the laser-active components in the supersonic turbulent flow of a CO_2 laser with mixing was taken into account in [26] using the DFPD of the concentration. However, the parameter characterizing the degree of mixing was determined on the basis of information on the single-point moments of the concentration field and taken into account only in the kinetic equations, which were solved separately from the one-dimensional gas-dynamic equations. The approach described in the present work allows the fact that the turbulent velocity and concentration fields have multiple scales to a significant extent to be taken into account, as well as their influence on the mean gas-dynamic and relaxational characteristics of the flow.

On the basis of the above approach, the characteristics of vibrationally-nonequilibrium CO_2 and N_2 flows mixing behind the nozzle array are calculated. The initial parameters of the flows before mixing are as follows: $r_1{}^0 = 0.171$ cm; $r_2{}^0 = 0.372$ cm; $u_1{}^0 = 619$ m/sec; $u_2{}^0 = 1880$ m/sec; $T_1 = 64$ K; $T_2 = 355$ K; $p = 1040$ Pa. The system in Eqs. (18)-(19) is solved with initial conditions for $P_0(r)$ and $P_0{}^c(r)$ taken from [18] and constants $\gamma = 0.07$, $\beta = 0.01$. The dispersion of the initial distributions $P_0(r)$ and $P_0{}^c(r)$ is $\sigma_u = 0.5$, $\sigma_c = 1.5$, respectively. The turbulent Reynolds number, calculated from the scale $\ell = 4r_1{}^0r_2{}^0/(r_1{}^0 + r_2{}^0) = 0.476$ cm, is $R_\ell = \sqrt{\overline{u'^2}(0)}\ \ell/\nu = 3095$. The functions $\varphi(x)$ and $\psi(x)$ obtained from Eqs. (32) and (21), taking account of Eq. (22), are substituted into Eqs. (11)-(17). Thus, they are taken into account as external information in the relaxational equations and the equations for the mean gas-dynamic quantities. The results of calculating the mean (in the direction of the beam) amplification factor $\bar{\alpha}$ and comparison with experimental results [27] (Fig. 3a) reveal not only qualitative but good quantitative agreement, which confirms the correctness of using the given approach to describe the turbulent mixing of relaxing gases behind the array of small-scale nozzles. The variation in φ and ψ with respect to the coordinate x is shown in Fig. 3b. It is evident that, when the proportion of mixed components reaches the level $\varphi \sim 0.4$, the intensity of pulsations of the turbulent velocity field is $\psi \sim 0.1$ and further mixing is determined basically by the slow process of laminar diffusion. Small values of γ and β in comparison with subsonic flow behind a small-scale array with large Reynolds numbers [18] indicate fast laminarization of the supersonic flow.

Note that, in the present work, the influence of variability of the mean density and the density pulsations on the gas-dynamic and turbulent characteristics of the flow was ignored. Taking this influence into account would evidently allow more accurate values of γ and β to be determined.

NOTATION

ρ, density; u, velocity; T, temperature; P, pressure; H, specific enthalpy; μ, molecular weight; e_j, mean number of vibrational quanta of j-th vibrational mode; ξ_k, molar concentration of k-th type of molecules; G_i, mass flow rate of i-th component in a state of mixing to the molecular level; $G_i{}^0$, total mass flow rate of i-th component; N_J, number of nonequilibrium vibrational modes in mixture; g_j, Θ_j, multiplicity of degeneracy and characteristic vibrational temperature of the given vibrational level of the j-th mode; $\bar{\alpha}$, mean (in the direction of the beam) amplification factor of medium; $P_t(r)$, $P_t^c(r)$, distribution of intensity of pulsations of the turbulent velocity and concentration field, respectively, with respect to the spectrum of scales of length r; ν, χ, kinematic viscosity and diffusion coefficient; L_m, U, T_m, characteristic values of the length, velocity, and time scales, respectively, of the mean motion; r_0, τ, characteristic values of length and time scales, respectively, of small-scale turbuelnt motion; x, longitudinal coordinate; t, time; r_i^0, initial linear dimension of jet of i-th component obtained after calculating the shock-wave interaction and additional expansion of the flows close to the plane of the nozzle grid; c, $\bar{c}$, c', instantaneous, mean, and pulsational value of the absolute concentration of the component in the flow, respectively; $f_t(c)$, density function of probability distribution of the concentration field; φ, proportion of components mixed to molecular level; ψ, intensity of turbulent velocity pulsations, normalized to the initial value $\overline{u'^2}(0)$. Indices: i = 1, 2, number of component being mixed; j = 1, 2, ..., N_J, number of nonequilibrium vibrational mode; L, N, linear and nonlinear molecules, respectively; superscript 0, initial value of parameter at the array plane.

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VELOCITY AND TEMPERATURE FLUCTUATIONS IN A TURBULENT SUSPENSION

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The effect of the particles of a suspension on the spectrum of velocity and temperature fluctuations is studied on the basis of the equations for the second two-point moments.

The effect of the particles on the spectrum of velocity and temperature fluctuations of a gas with particles arises in connection with the propagation of acoustic, optical, and radio waves in a dusty medium. The distortion of the spectrum of fluctuations of the gas component due to the particles has not been studied sufficiently either theoretically or experimentally. There is no data in the literature on the spectrum of temperature fluctuations of a gas with particles and there is no common view on the nature and degree of the distortion of the distribution of fluctuation energy of the gas among vortices of different sizes in a suspension. For example, it is assumed in [1] that the addition of particles into a turbulent fluid does not change the intensity of velocity fluctuations of power-consuming vortices but leads to a suppression of small-scale vortices whose characteristic sizes are smaller than the diameter of the particles suspended in the fluid. The model of [1] was applied in [2, 3] to the hydrodynamics and heat transfer of the flow of a suspension in a pipe. In [4] the spectrum of velocity fluctuations of the gas component of a suspension was studied theoretically and it was found that the intensity of turbulent velocity fluctuations increases in the inertial part of the spectrum and decreases in the region of viscous dissipation. But the theoretical picture of the distortion of the spectrum of velocity fluctuations of the gas in the presence of particles does not agree with the experimental data of [5, 6]. In these papers it was established that small particles lead to a significant decrease in the intensity of turbulent velocity fluctuations of the gas in energy-containing vortices and in the inertial region of the spectrum, while in the viscous dissipation region fluctuations increase.

In the present paper we consider a fluid with a small volume concentration of impurity particles. On the basis of the equations for the second two-point correlations of the velocity and temperature fluctuations in the discrete and fluid phases we obtain expressions for the spectral functions describing the intensity distribution of velocity and temperature fluctuations of the gas phase as functions of the wave number in the inertial and convective regions of the spectrum. We study the effect of the ratio of the heat capacities of the particles and the gas and also the molecular Prandtl number of the gas on the spectrum of temperature fluctuations of a gas with particles.

The system of equations for the second two-point correlations of the velocity fluctuations for the fluid and discrete phases has the following form, assuming homogeneous isotropic turbulence [4]

$$\frac{\partial E_u(k,t)}{\partial t} - F_u(k,\ t) = -2\nu k^2 E_u(k,\ t) - 2\frac{\Phi}{\tau_u}[E_u(k,\ t) - E_{uv}(k,\ t)], \tag{1}$$

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